A model for stress-controlled pipe growth

Wayne P. Barnett a,⁎, Loren Lorig b

Lithosphere Dynamics Group, De Beers, South Africa
Itasca South America, Santiago, Chile

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Abstract

The rock mechanics theory for deformation of underground mining excavations under high stress conditions can be used to explain the growth and geometry of volcanic pipes. In an underground excavation stress concentrates greatest on the sides of an excavation perpendicular to the principal vector of compression. If the stress is high enough fractures will develop causing scaling of the tunnel sidewalls and tunnel growth perpendicular to the principal vector of compression. Pre-existing structures aid the physical mechanisms of pipe growth such as gravitational collapse, explosive fragmentation and turbulent erosion; and by reducing the strength of the rock mass should also aid stress-induced scaling. Universal Distinct Element Code numerical modelling demonstrated in this study reproduces the stress conditions around a circular pipe under uniaxial compression and simulates pipe growth as wedges bounded by failed pre-existing joints form around the pipe and are “assimilated” into the pipe. The results show how a volcanic pipe will tend to grow perpendicular to the principal vector of compression if the internal magma pressure is low or absent. The orientations of the pre-existing joints affect the exact direction of pipe growth in a predictable manner. Examples from other publications demonstrate that the model is consistent even in extensional tectonic environments. Case studies from kimberlite occurrences in the Limpopo Belt, at Finsch Mine and the Gross Brukkaros Volcanic Complex demonstrate that dykes and magmatic bodies of kimberlite with high overpressures during emplacement normally have geometries trending near parallel to the principal vector of compression. Yet pipes or parts of pipes that underwent strong underpressures during emplacement (often ending up comprising fragmental volcaniclastic infill) have an elongation near perpendicular to the same vector. Thus the stress-induced pipe growth model is demonstrated to be important for some pipes. The study of volcanic pipe and dyke shapes can therefore be used to determine the stress regime at the time of emplacement, and to distinguish between kimberlite occurrences formed at different times within different stress tensors.

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1. Introduction

1.1. Excavation growth mechanisms

Although it is not a well understood and studied aspect of volcanology, there are a number of physical processes that contribute to the excavation of a volcanic pipe and cause the horizontal expansion of a sub-vertical pipe. We use the word “pipe” loosely in this paper to refer to any part of the volcanic supply conduit, root zone, diatreme and subsurface crater; as long as it remains an “open” magma conduit even when the driving magma pressure is removed.

Gravity is an obviously important physical process that aids excavation. Gravitational stoping has been used to explain the growth of large magmatic bodies,
and has been used to explain the formation of ring faults and contact breccias found around volcanic pipes (Hearn, 1968; Clement, 1982; Field and Scott Smith, 1999; Kurszlaukis and Barnett, 2003; Barnett, 2004). Gravitation slumping has been better studied within the crater environment because of the better accessible exposures (e.g. Belousov and Belousova, 2000; Koncény and Lexa, 2000). It is also the primary cause of the formation of calderas above volcanic magma chambers (Acocella et al., 2004), often in combination with ring faults.

A volcanic explosion is a common physical excavation process, induced either by a build up and subsequent violent release of gaseous pressure (Clement, 1982; McCallum, 1985; Field and Scott Smith, 1999; Wallace and Anderson, 2000), or by hydro-magmatic explosions induced by direct coupling of well-mixed water and magma resulting in a violent exothermic reaction (Lorenz, 1975; Büttner and Zimanowski, 1998; Kurszlaukis et al., 1998; Lorenz et al., 1999). Such explosions can cause the pipe to grow horizontally and/or vertically in size, and, in the case of a volcanic pipe dominated by phreatomagmatic processes, these explosions account for the downward growth of the diatreme into the feeder-pipe (Lorenz, 1985).

Turbulent erosion has also been used to explain the growth of a kimberlite pipe (Clement, 1982; McCallum, 1985). Turbulence can be generated by the energetic release of gaseous phases from the magma and the resulting fluidization of magma and pyroclastics within the pipe.

The potential for stress-induced scaling (slivers or scales of rock fall into the pipe) of a volcanic excavation as a growth mechanism is often overlooked. Adamović and Coubal (1999) use the intrusive geometries of Late Cretaceous to Pliocene volcanic bodies in the northern parts of the Bohemian Massive to refine an existing paleostress model for the area. The K/Ar dates for the intrusive bodies range from 77 Ma to 9 Ma and the time progressive variation in the intrusive body orientations matches and further refines the expected variation in the tectonic stress tensor over that time period. In order to do this, Adamović and Coubal (1999) use the models of magma emplacement formulated by Pollard (1973) with regards to dyke emplacement, and further note that the elongation of “pipelike” non-tabular bodies tended to be parallel to the minimum principal component of regional stress. They speculate that the pipe elongation is a result of elastic strain (pure shear) of the country rock effectively “squashing” the pipe, but concede that this does not explain the large sizes and eccentricities of the plugs.

In this document a pipe growth model is proposed suggesting that the propagation direction of a dyke and the growth direction of a volcanic excavation through stress-induced scaling or slabbing (joint bounded rock slabs fall into the pipe) should theoretically be at 90° to each other, where slabbing is approximately perpendicular to the principal vector of compression ($\sigma_1$). The exact direction may be strongly influenced by pre-existing structures (Delaney et al., 1986). In the case of kimberlites it could be assumed that hypabyssal/magmatic bodies with a high overpressure follow the emplacement rules of dykes (preferentially parallel to $\sigma_1$; Pollard, 1987), and that pipes/cavities with underpressure (often later filled by volcaniclastic facies) may follow the growth rules of an excavation (perpendicular to $\sigma_1$). This report describes numerical modelling that has been utilized to demonstrate the slabbing growth process and the influence of pre-existing structures on such a process. The results of the model are then applied to the Limpopo Belt kimberlites, the Finsch kimberlite pipe and the Gross Brukkaro carbonatite complex as case studies.

1.2. Fracture growth and stress-induced scaling

In rock mechanics the degradation of an underground excavation is well understood, measured and analyzed (e.g. Budavari, 1983). The reduced strength of a rock mass due to internal fractures and fluid pressures can be empirically quantified (e.g. Laubscher, 1990). Before an excavation is formed within the rock, the principal stresses are at equilibrium. Once rock is removed to form the excavation, the stresses applied to the rock immediately on the boundary of the excavation become zero and must remain zero, and the equilibrium is therefore destroyed. In order to re-establish equilibrium, the stresses then redistribute around the excavation in a manner depending on the geometry of the excavation. Such re-equilibration is not instantaneous. The elastic response of the rock should be very fast (speed of wave propagation in the rock), but relatively high stresses compared to the rock mass strength will induce fracture propagation and shearing of planes of weakness with a resultant slow plastic-like deformation of the entire rock mass. The slow “viscous” behaviour of such strain takes a longer time to equilibrate.

In mining, three levels of conditions are often referred to and essentially equate to the typical stress conditions at specific depth ranges. Shallow level mining occurs above 1000 m below surface and is typified by tensile rock mass behaviour with strong structural control on rock movement. Intermediate level mining occurs between 1000 m and 2500 m and problematic strain induces high stress concentrations and associated fracturing or rock burst risks, and/or movement on pre-existing structures.
depending on the situation. Deep level mining, below 2500 m, induces high stress compressive deformation (fracturing and rock bursts), but structures tend to be too tightly confined to cause failure problems. These level categories are used as guidelines only and can be very different when dealing with specific mining, topographic or tectonically induced stresses. When an excavation is located within a relatively highly stressed rock, stress concentrates around the excavation and causes fracture propagation (Budavari, 1983).

Tensile fractures grow parallel to the direction of the greatest (compressive) principal stress vector \( \sigma_1 \), and lie parallel to the plane defined by the maximum \( \sigma_1 \) and intermediate \( \sigma_2 \) principal vectors, and therefore perpendicular to the minimum compressive principal stress vector \( \sigma_3 \). As a simplified example, a perfectly circular horizontal tunnel analyzed under plane-strain and purely elastic conditions undergoing vertical uniaxial compressive stress \( q \) has a simple analytical solution (see the Kirsch equations; e.g. Budavari, 1983) at the point of re-equilibrium (Fig. 1a). On the tunnel boundary the radial stress is 0; but the tangential stress is equal to \(-q\) on the top of the tunnel \( (\theta=0^\circ) \), and equal to \(3q\) on its sides \( (\theta=90^\circ) \). Fractures near the tunnel will therefore grow parallel to the tangential stress (i.e. circular around the tunnel), but are much more likely to form on the sides of the tunnel \( (\theta=90^\circ) \). This means that the sides of an excavation that are closest to parallel to \( \sigma_1 \) undergo greater scaling (slivers or scales of rock fall into the tunnel) causing excavation growth perpendicular to \( \sigma_1 \) (Fig. 1b). In practice, this effect is used to estimate stress directions reliably when observed around tunnels, boreholes and vertical shafts. Highly stressed volcanic pipes should undergo similar effects.

The tunnel in Fig. 1a could be rotated to represent a vertical volcanic pipe but filled with magma \( (p \neq 0) \). Internal fluid and magma pressures \( (p) \) change the simplified uniaxial stress equations for the tunnel discussed above (Fig. 1a) to \(-q-p\) and \(3q-p\) for \( \theta=0^\circ \) and \( 90^\circ \), respectively. As magma pressure \( (p) \) increases, the tangential stress at \( \theta=90^\circ \) reduces and fracture growth is less likely. If \( p \) increases above \(-q\) then a tensile stress forms at \( \theta=0^\circ \), and if \( p \) is high enough to overcome the tensile strength of the rock then the fracture/dyke propagates parallel to \( \sigma_1 \) (Fig. 1c).

In this simple mathematical formulation lies the difference between tensile/hydraulic fracture propagation parallel to \( \sigma_1 \) (Fig. 1c), static equilibrium, and fracture propagation and scaling causing excavation growth perpendicular to \( \sigma_1 \) (Fig. 1b). Note that a dyke is usually tabular in shape, but for the purpose of this discussion we treat a volcanic excavation (a pipe as defined above) with a high internal magma pressure driving pipe growth as a feeder dyke, with the behaviour of a typical dyke.

1.3. Structural controls and explosive volcanism

The physical growth processes discussed above are all strongly influenced by pre-existing rock discontinuities (fractures, joints, faults, etc.). Such structures reduce the overall rock mass strength (Beniawski, 1978) and allow gravitation, explosive forces, turbulent erosion and rock stresses to be more effective as pipe growth mechanisms. Dykes are understood to develop in the same way that a fracture propagates parallel to \( \sigma_1 \) (as explained above; Fig. 1c; Pollard, 1987), particularly if the difference in magnitude of the principal stresses is

Fig. 1. a) Schematic section illustrating stress distribution around a circular tunnel in vertical uniaxial stress conditions of magnitude \( q \). Modified after Budavari (1983). \( st \) and \( sr \) represent the tangential stress and radial stress respectively. b) Schematic section of a volcanic pipe undergoing stress-induced scaling perpendicular to the maximum compressive stress \( (\sigma_1) \) in the plane of the section. The mean stress \( (q) \) is greater than the internal magma pressure \( (p) \). The tunnel will widen through a process of scaling as the fractured slivers fall into the tunnel. c) Schematic section of a volcanic pipe with a high magma pressure \( (p) \) relative to the mean stress \( (q) \). Hydraulic dyke and fracture propagation will occur parallel to the maximum compressive stress \( (\sigma_1) \) in the plane of the section.
high (Delaney et al., 1986). If the difference in magnitude between the principal stresses is low and the magma pressure is a significant driving emplacement force then the dyke will utilize any pre-existing structures that are orientated at an angle that allows the structures a tensile component of displacement (Delaney et al., 1986). In extreme cases of high magma pressure, the dyke may intrude a structure orientated in any direction. In summary, a dyke is not always orientated parallel to \( \sigma_1 \) because of preferred use of pre-existing structures.

The influence of pre-existing structure on the stress-induced scaling process in a volcanic excavation should also be analyzed. The example of high stress around a tunnel with the initiation of fracture growth was discussed above and is also mathematically derived. Growth should be perpendicular to \( \sigma_1 \), but as in the case of dykes, pre-existing structures should also play a role in tunnel growth, particularly at lower stresses. When considering structures around a tunnel or pipe the stress energy is not required to fracture the intact rock, but only to overcome the cohesive and frictional forces on the pre-existing structures. Cohesion is a property of the rock that must be overcome for a potential failure surface to undergo shear displacement. It is typically in the order of kilopascals (kPa, thousands of pascals), whereas rock tensile and compressive strength as well as in situ and tectonic stresses are measured in megapascals (MPa, millions of pascals). Stress-induced scaling (or rather slabbing – rock slabs fall into the excavation) should therefore occur at lower stresses by utilizing such pre-existing structures. Slabbing could potentially become the dominant pipe growth mechanism around pipes with highly fractured wall rock, and the process is modelled in the next section.

It is worth noting that an explosive pipe growth mechanism is very different to slabbing. An explosion induces a radial outward directed (hydraulic) stress that induces radial fractures. This explosive stress (high overpressure) is the driving mechanism, but can also be strongly influenced by pre-existing structures and in situ stress. Again, this is a fairly well understood and observed process in mines in blasting engineering. The blasting fractures will grow longer in the direction of \( \sigma_1 \), such as in the case of a dyke. Structures closer in orientation to \( \sigma_1 \) will also be preferentially opened (destroying cohesion on the structure) and utilized for gas expulsion. In this way a pipe growth process dominated by phreatomagmatic explosions should theoretically grow parallel to \( \sigma_1 \), and be strongly influenced by structural fabric in the country rock. This would also be the most likely direction of water influx if that influx is structurally controlled. An excellent example is the Quaternary Westeifel volcanic field in the Hercynian Rhenish Massive in Germany (Büchel, 1984), where the vent alignments, maar shapes, dykes and controlling structures trend predominantly parallel to the \( \sigma_1 \) NW–SE direction or to a lesser degree the conjugate shear planes +40° and −40° to \( \sigma_1 \). Explosive pipe growth processes are not modelled in this paper.

2. Pipe growth modelling

2.1. UDEC numerical model

It is the intention of this part of the study to numerically model the slabbing growth process to demonstrate that jointed rock would produce similar excavation growth as scaling, and to see the influence of joint orientation on the growth direction. A variety of stress modelling packages are available. Universal Distinct Element Code (UDEC) is an HCItasca Pty (Ltd) product that is particularly suited to modelling discontinuous media such as jointed rock masses (De Lemos, 1987). It is a two dimensional modelling program and uses the distinct element method of discontinuum modelling, in which the medium is treated as an assemblage of discrete blocks with the discontinuities forming boundary conditions between the blocks. UDEC allows rotation of blocks relative to adjacent blocks, and allows large displacements on the discontinuities. It automatically recognizes complete detachment of blocks as well as new contacts. Each block can be made deformable by dividing it into finite-difference elements that respond to a user-specified stress–strain law. The block boundaries (discontinuities) also behave according to a user-specified normal and shear force–displacement law.

The model created for the present pipe growth analysis is 1000×1000 units in size (Fig. 2). For each analysis the model is cut by a user specified selection of cracks and joints. Each joint-bounded block is made deformable in order to transmit elastic stress realistically through the model. Before boundary stresses are applied, a circular excavation (hole) is created in the centre of the model that is 80 units in radius and the model is allowed to equilibrate (“settle”) with an internal stress. It was not the intention of this analysis to model actual rock and joint properties, but rather to observe the behaviour of joints around the excavation if the joints are arbitrarily weaker than the intact rock mass. A compressional stress boundary condition was applied to the “east” and “west” sides of the model, where these boundaries move towards each other at a fixed velocity. The “north” and “south” boundary of the model was fixed as a roller-boundary, allowing the “east–west” movement but not allowing
block escape to the “north” or “south”. In this way the stress continues to increase throughout the run-time period of the model. Measurements of actual stress magnitudes are not the objective of the study. It is essentially a uniaxial compression model with infinite lateral confinement, providing a good approximation to local stress and strain conditions in an area undergoing compressional tectonics. The properties of the rock, the joints and stiffness between blocks were selected simply to allow reasonable model runtimes. The rock properties used were a density of 2.84 kg/m³, a bulk modulus of 32 GPa and a shear modulus of 25 GPa. The discontinuity failure criterion used was a simple Coulomb failure criterion with joint cohesion of 0 kPa, joint tensile strength of 0 kPa and friction angle of 25°.

A number of assumptions are then made in the model. The perfectly circular hole in the centre is regarded as an initial volcanic pipe crosscut in plan view, within which there is no magma pressure. As the applied stresses increases, the discontinuities around the tunnel fail either in tension or in shear (they slip). If such failed joints link up to form a failed “wedge” (an assemblage of UDEC blocks in any shape that is completely surrounded by failed joints) that shares one boundary with the excavation, then that block is assumed to be removed by volcanic processes (e.g. gravity) and the excavation expands into the area in which the wedge was located. The volcanic pipe then grows by repeated formation and removal of wedges. This is representative of the slabbing process discussed above.

UDEC does not identify and remove the wedges automatically. A programming language called FISH is available to control the models as required and can be used to undertake automated queries of the model, such as the detection of wedges. A FISH algorithm called “Slab” was programmed that does the detection and deletion of the required wedges.

For the purposes of the modelling and the discussion through the rest of this document, the direction of “east–west” compression in Fig. 2 is the orientation of the principal vector of maximum compression ($\sigma_1$). The “north–south” direction in Fig. 2 is regarded as the principal vector of minimum compression ($\sigma_3$). The problem is therefore regarded in this document as a purely two-dimensional problem in plan view, and the magnitude of the vertical stress is not considered. The primary effect of the vertical stress should not be to affect the direction of pipe growth, but rather the rate of pipe growth. An excavation near the surface will have a low vertical stress that has very little effect on the growth model. An excavation at greater depth will have a higher vertical stress that contributes to the horizontal stress equally in all directions, and in magnitude as a function of Poisson’s ratio. However, it also provides greater confinement on most structures. The modelling discussed below is only concerned with the direction of pipe growth.

2.2. Modelling results

A range of joint patterns were modelled, including two (Fig. 3), three (Fig. 4) or four joint sets (Fig. 3f). In all cases the first growth direction is close to perpendicular to the compression direction, confirming the discussion above that joints located at $\theta = 90^\circ$ and $-90^\circ$ to $\sigma_1$ around an excavation will experience the greater stress and fail first. The relative orientation of the pre-existing structures does play a role in the pipe growth direction. Specific models are now discussed. In all the models a polar co-ordinate system is used to indicate direction, where $\theta = 0^\circ$ is parallel to $\sigma_1$ and on the left (“west”) of the excavation. $\theta = 90^\circ$ is parallel to $\sigma_3$ and clockwise from $\theta = 0^\circ$.

2.2.1. Two joint sets

In models with two joint sets (Fig. 3a,b,c,d,e) a wedge is quickly formed creating pipe growth in a direction parallel to an imaginary bisector of the two joint set strikes – that bisector that is closest to perpendicular to $\sigma_1$. The model in Fig. 3a most clearly...
shows the stress distribution and the growth direction. The model in Fig. 3b demonstrates the growth model least clearly, since the “north–south” orientated joint set is under pure compression and does not fail, which means that wedges cannot form and pipe growth cannot initiate. Eventually the second set does fail under shear as the areas around \( \theta=45^\circ, 135^\circ, 225^\circ \) and \( 315^\circ \) develop high tangential stresses. The wedges formed cause the pipe to grow into a perfect square. A common result is evident: pipe growth is still parallel to a bisector of the two joint set strikes.

The models in Fig. 3b,c,d illustrate another phenomenon. Failed joints initiate at the \( \theta=90^\circ \) and \(-90^\circ\) positions and extend along the joint set most parallel to \( \sigma_1 \) as expected from rock mechanics studies. These failed joints extend a considerable distance away from the pipe excavation. Occasionally two such parallel failed joints join by shearing across a cross-cutting joint and a wedge is created as defined above. The wedge is long and thin and strictly should not be incorporated into the pipe, but the code does not distinguish a difference and the thin wedge is deleted (e.g. Fig. 3b,c,d). Note the geometrical similarity between these thin wedges and dykes growing from the pipe edge. One must be careful not to create a real geological phenomenon to explain the manifestation of an inadequacy in a computer algorithm. However, it is worth speculating on how similar an actual dyke process could be to the observation in the model. It seems likely that during times of high magma pressure and resurgence, syn-volcanic dykes could most easily utilize the failed joints that have grown at the edges of the pipe and thereby radiate away from the pipe in identical fashion to the models. Syn-volcanic dykes, fissures and vents have commonly been observed radiating from volcanic pipe centers preferentially in the direction of \( \sigma_1 \) (Bosworth et al., 2000; Nakamura, 1977). In kimberlite geology, the dyke and pipe relationship is often interpreted as a precursor feeder dyke with a pipe “blow” centered on the dyke. Some of these dykes could rather be re-interpreted as above.

2.2.3. Three joint sets

A similar guideline of growth direction can be created for models with three joint sets. Each set adds to an aggregated total influence on pipe growth. It appears to be important to determine which sets are undergoing dextral shear, and which are being sheared sinistrally. If one dextral set is combined with one sinistral set we can call the combination a conjugate set. With a total of three joint sets it should be possible to identify two possible conjugate sets, where one joint set is used in both conjugate sets. As in the case of the models with just 2 joint sets, the imaginary bisector of a conjugate set (that is closest to being perpendicular to \( \sigma_1 \)) will be a direction of pipe growth. With two possible conjugate sets there are two imaginary bisectors, and two directions of growth. From the results of the modelling, it appears that each bisector has a near equal influence on the resultant growth direction. The resultant growth is therefore the average, or bisector of the bisectors.

More work can be done to confirm the above guideline. The relative frequency of one set compared to another is likely to have an effect in reality, and individual large structural features that undergo significantly more strain than local minor structures will distort the stress field. It should be clear that the more joint sets available for pipe growth the more likely that the pipe will grow perpendicular to \( \sigma_1 \).

2.3. Extensional tectonic regimes

The above numerical simulation applied compressional tectonics to a volcanic pipe. To have a full understanding of pipe growth and to apply models in any tectonic environment, the mechanisms also needs to be considered in extensional tectonic regimes. Realistic pipe growth has not been modelled in extensional conditions using UDEC. However, Arocella et al. (2004) describe a physical sand-box model experiment in which the volcanic calderas and domes above a magma chamber are modelled undergoing collapse and resurgence, respectively. They specifically tested to see the interaction of structures formed by collapse and resurgence with pre-existing extensional structures. The authors conclude that magma withdrawal with caldera collapse produces an elliptical shaped depression orientated with the major axis parallel to the direction of extension \( \sigma_3 \), because of the interaction with the pre-existing structures. The ellipticity (minor axis/major axis) decreases in value (becomes more elliptical) by 0.1 for every pre-existing fault with which it interacts.

A comprehensive case study was undertaken by Bosworth et al. (2000) on the shield volcanoes of the...
Fig. 3. UDEC illustration of modelling results. The continuous solid grey lines represent joints, which are black where they have failed and undergone displacement. Short black lines represent the local orientation of the $\sigma_1$ vector with the length proportional to the magnitude of the stress. They thereby illustrate the effect of the pipe on the stress field. All models show pipe growth from stress induced failure of joints, where the growth tends to be perpendicular to $\sigma_1$. Note the concentration of greatest compressive stress on the “north and south” edges of the pipe. Each model’s joints differ as follows: a) Two joint sets angled $+45^0$ and $-45^0$ to $\sigma_1$. b) Two joint sets angled 00 and 900 to $\sigma_1$. c) Two joint sets angled $+67^0$ and $-23^0$ to $\sigma_1$. d) Two joint sets angled $+80^0$ and 00 to $\sigma_1$. e) Two joint sets angled $+180$ and $-180$ to $\sigma_1$. f) Four joint sets angled $+180$, $+720$, $-180$ and $-720$ to $\sigma_1$.

Fig. 4. UDEC illustration of modelling results for models containing three joint sets. All models were subjected to “east-west” compression as illustrated by arrows in (a). The continuous solid grey lines represent joints, which are black where they have failed and undergone displacement. Short black lines represent the local orientation of the $\sigma_1$ vector with the length proportional to the magnitude of the stress. They thereby illustrate the effect of the pipe on the stress field. All models show pipe growth from stress induced failure of joints, where the growth tends to be perpendicular to $\sigma_1$. Note the concentration of greatest compressive stress on the “north and south” edges of the pipe. The grey shading in (d) represent contours of strain (strain in the x-axis or “east-west” direction) around a pipe as it undergoes compression. The contoured strain values increase during the run-time of the model. Each model’s joints differ as follows: a) Three joint sets angled $+800$, 00 and $-400$ to $\sigma_1$. b) Three joint sets angled $+100$, 00 and $+400$ to $\sigma_1$. c) Three joint sets angled $+400$, 100 and $-400$ to $\sigma_1$. d) Three joint sets angled $+800$, $+400$ and $-400$ to $\sigma_1$. e) Three joint sets angled $+180$, $+720$, $-180$ and $-720$ to $\sigma_1$. f) Four joint sets angled $+180$, $+720$, $-180$ and $-720$ to $\sigma_1$. f) Four joint sets angled $+180$, $+720$, $-180$ and $-720$ to $\sigma_1$. f) Four joint sets angled $+180$, $+720$, $-180$ and $-720$ to $\sigma_1$. f) Four joint sets angled $+180$, $+720$, $-180$ and $-720$ to $\sigma_1$. f) Four joint sets angled $+180$, $+720$, $-180$ and $-720$ to $\sigma_1$. 

Kenya Rift Valley (Late Pleistocene to Recent in age) that supports the concept of stress-induced scaling of volcanic excavations. The stress direction changes over the life of the volcanic field are easily reconstructed from the alignment of fissures, faults and volcanic cones all parallel to the rift margins, i.e. perpendicular to the rifting extension direction $\sigma_3$. They assume that the strongest control on dyke orientation is the country rock stress field, and that surface vents are also generally aligned parallel to $\sigma_1$. The above, combined with fault kinematic studies, suggests that the rift valley underwent E–W extension during the late Pliocene to Early Pleistocene. Present-day borehole breakout (stress-induced scaling similar to a tunnel) studies and aligned fissures, vents and cone trends younger than ca. 125 ka indicate a fairly recent and progressive stress rotation with $\sigma_3$ aligned WNW and later NW–SE (Fig. 5). The authors note that the caldera elongations above collapsed magma chambers closely reflect the above stresses and stress changes.

Unlike the fissure, vent and cone trends, the calderas are elongated parallel to $\sigma_3$. Bosworth et al. (2000) explain this as a result of magma chamber elongation parallel to $\sigma_3$ in the same way tunnels fracture, slab and grow. Such a situation can only occur at stages when the magma pressure drops allowing stress fracturing around the excavation. When the magma withdraws from the chamber the induced collapse caldera above the chamber mimics the chamber in elongation. It can therefore be demonstrated that the model of volcanic pipe growth parallel to $\sigma_3$ and perpendicular to $\sigma_1$ due to stress-induced slabbing holds in both extensional and compressional tectonic environments.

### 3. Case studies

In applying the pipe growth model discussed above to real case examples it is ideally necessary to know whether the pipe formed in conditions of high magmatic pressures (overpressure relative to the tectonic stress) or in conditions of low magmatic pressure (under pressure). This information is not likely to be known, and in reality a pipe might undergo various phases of both under- and overpressure. Derived paleostress models are therefore speculative if not supported by additional information. However, the following suggestion can be useful. Long, thin intrusive bodies dominated by hypabyssal rock are more likely to have grown in overpressure conditions. Wider, more oval intrusive bodies dominated by fragmental volcaniclastic rock are more likely to have experienced underpressure conditions with stress-induced slabbing.

#### 3.1. Case study 1: Application of the pipe growth model to the Limpopo Belt kimberlites

##### 3.1.1. Venetia

The Limpopo Belt is a late Achaean to Early Proterozoic orogenic belt separating the Zimbabwean craton from the Kaapvaal craton in southern Africa. The Venetia kimberlites are located in the Limpopo Belt about 25 km south of the three-way junction between the Botswana, Zimbabwe and South African border (Fig. 6). The kimberlites are believed to have been emplaced at 519 Ma (Phillips et al., 1999). The outlines of a tectonic stress model for the emplacement of the Venetia kimberlite cluster is discussed in Kurszlaukis and Barnett (2003). The configuration of structures suggested approximately N–S compressional and relative E–W extension as a most likely stress model. Such a

![Fig. 5. Schematic illustration of the tectonic stress direction change and resulting effect on caldera shapes in the East African Rift System. Modified from Bosworth et al. (2000).](image-url)
model is plausible since emplacement is coeval with metamorphic ages in northern Zimbabwe associated with broadly NNW–SSE oriented \( \sigma_1 \) stress of the Pan-African Lusilían orogeny (Master and Kramers, 2000; Johnson et al., 2005).

Fig. 7 illustrates a slightly revised emplacement model, in which the cluster is located in a concentration of N–S to NNE–SSW striking joints. These joints form an accommodation zone in which localized zones of crustal extension could be found (Fig. 7, inset). Sinistral reactivation of the Lezel fault would have produced a dilation jog through which the initial volcanic feeder dyke may have intruded. The best fit local \( \sigma_1 \) stress orientation is NNE–SSW. The Venetia kimberlite cluster and possible emplacement stresses are now reviewed in context of the pipe growth model presented above.

Consider firstly the primary magmatic occurrences that should trend parallel to \( \sigma_1 \) (ca. NNE–SSW) in the pipe growth model. Few true hypabyssal dykes have been mapped around the Venetia pipes. One known dyke strikes in a southwest direction, but it is also located near the Lezel fault which has been interpreted to have been active during emplacement. Most of the magmatic component of the K1 pipe is found in the eastern “foot-shaped” side of the K1 pipe. The area of the pipe is located on a dilation jog of the Lezel fault, which would be an ideal area of extension in which the original intrusion would find the least stress resistance. This “foot-shaped” edge of the pipe trends north–northeast.

The smaller pipes east of K1 (i.e. K16, K10, K6) and connecting dykes are strongly weathered and in some cases sheared, but are likely to be magmatic kimberlite. They appear to have intruded the Lezel fault and splays, and strike between N–S and NE–SW. K8 is a large hypabyssal breccia pipe west of K1, which strikes NNE–SSW. This pipe would seem to be an excellent indication of \( \sigma_1 \) orientation.

The large country rock breccia body extending southwards from the western side of K1 is slightly more difficult to explain (Fig. 7). Barnett (2004) shows by means of detailed fractal studies on breccia clast sizes and shapes that the breccia formed through at least two phases of fragmentation. The first phase appears to have been
explosive fragmentation, possibly phreatomagmatic. The second phase has been interpreted as a subsidence driven fragmentation and shearing, as the breccia gravity-fed into the K1 pipe. Kurszlaukis and Barnett (2003) present textural and petrographic evidence that phreatomagmatic processes did occur in the Venetia kimberlite pipes, at least during certain time intervals of volcanism. Given the proposed stress model an explosively derived breccia would cause local pipe growth towards the SSW, parallel to $\sigma_1$ as is observed (Fig. 7).

Now consider the Venetia pipes/facies that are filled with fragmental volcaniclastics. In the pipe growth model the elongation of these pipes/facies would be expected to be orientated orthogonally to $\sigma_1$ (ca. ESE–WNW). K1 and K4 are clearly orientated in a WNW direction. The longest axes of K2, K3 and K5 are also orientated in an E–W direction. All of these pipes are dominated by fragmental volcaniclastics, with possible exception of K4 (Colgan, 1982; Skinner, 2000). However, field observations of exposures in the upper benches of the K4 pit (Kurszlaukis and Barnett, 2003) suggest that a fragmental kimberlite dominates the upper K4 pipe and includes the presence of penetrative slickenside planes, large “floating reef” xenoliths and subsiding contact breccias.

We have exceptionally good evidence for the actual pipe growth process captured in situ on the WSW corner of K2. As described in detail in Kurszlaukis and Barnett (2003), this edge of the pipe forms a protrusion growing towards K3. The protrusion does not appear (according to on-site mapping) to continue vertically to the surface, rather forming a cave into which the K2 volcaniclastics...
have flowed, and onto which the cave hanging wall has later collapsed. This protrusion can be easily explained as a joint bounded wedge collapsing into the pipe as the pipe expands near orthogonally to $\sigma_1$ as per the pipe growth model. The gravitationally induced cavity was still in the process of stoping to the surface when it filled with volcaniclastic material. It is very likely that this represented the final phase of volcanism in K2 and the protrusion never had the opportunity to extend further vertically or laterally. All the above observations of pipe shapes support a stress model (Fig. 7) with maximum compression orientated NNE–SSW during Venetia kimberlite emplacement.

3.1.2. River ranch

The River Ranch kimberlite pipe is considered next in context of the pipe growth model. The River Ranch pipe is located within the Limpopo Belt 12.5 km WNW of the border town of Bietbridge in Zimbabwe, and 1.6 km north of the Limpopo River (Fig. 6). Muusha (1997) interpreted the pipe to be dominated by volcaniclastics. No reliable date is known for River Ranch, but U–Pb and Pb–Pb isotopes produced discrepant ages that are nevertheless similar to the Venetia kimberlites (Kramers and Smith, 1983).

The overall pipe shape is similar to Venetia K1 pipe (Fig. 8). The River Ranch subsidence breccias, described in Barnett (2004), occur precisely along the long axis of the pipe orientated E–W. The pipe appears to have been in the process of extending both eastwards and westwards approximately perpendicular to the proposed $\sigma_1$ direction (Fig. 8), as would be predicted by the pipe growth model. The main magmatic component of the pipe has a NNE to NE trend.
Assuming stress-induced pipe growth, a NNE–SSW oriented $\sigma_1$ stress model is supported.

3.1.3. The oaks

The Oaks kimberlite pipe is also located within the Limpopo Belt (Fig. 6), in South Africa near the town of Swartwater. It is very different in shape to Venetia K1 pipe and River Ranch pipe, but dated at a similar age of \( \sim 504 \) Ma (Phillips et al., 1999). Another difference is that no volcaniclastic kimberlite has been mapped at the Oaks (Mine Geologist T. Rowlands, personal communication), but the pipe is dominated by magmatic kimberlite and country rock breccias. The pipe is orientated towards the north–northeast (Fig. 9), in close agreement with the pipe growth model for magmatic pipes if the internal magma pressure was high, and very similar to the Venetia K8 pipe.

All the above pipe occurrences (Venetia, River Ranch and The Oaks) indicate a similar stress orientation for kimberlite emplacement in the Limpopo Belt at around 500 Ma and all imply NNE–SSW $\sigma_1$ orientation at that time.

3.2. Case study 2: The Finsch kimberlite pipe

The Finsch pipe comprises predominantly fragmental volcaniclastic facies (Clement, 1982) and is dated at $118 \pm 2.2$ Ma (Smith et al., 1985). It is located in South Africa about 180 km WNW of Kimberley near the town of Lime Acres (Fig. 6). Detailed country rock mapping (Barnett and Preece, 2002) identified four regional, sub-vertical joints sets developed in the country rock at Finsch Mine (J1 to J4; Fig. 10b) and kimberlite dyke zones external to the pipe (Fig. 10a). The dykes intrude two pre-existing joint sets (J1 and J3) striking NE and ENE, respectively (Fig. 10c). Only three joints sets are typically found together at one location, and J2 is much more dominant than J4.

The dyke orientations are fairly close to those similar aged kimberlite dykes in southern Africa described by Basson and Viola (2003). They conclude that the dykes are passively emplaced, meaning that the magma pressure was not great compared to the regional tectonic stresses, and that the kimberlite was allowed into the
pre-existing joints. If this is the case it should be an ideal scenario for stress-induced growth of a pipe.

The Finsch pipe shape does not have an obvious elongation at shallow depths, but at a depth below current surface of about 430 m the pipe starts to show a distinct NW–SE elongation, which is approximately perpendicular to the dyke trend (Fig. 10a). Of the three dominant sets, J1 is most parallel to local $\sigma_1$ and would probably have the least influence in deviating the pipe growth from being perpendicular to $\sigma_1$. The J2 and J3 sets’ bisector is parallel to $\sigma_3$ and to the pipe growth axis. The Finsch pipe has a number of protrusions located around the main diatreme. These protrusions are not late extensions on the pipe, but precursor intrusions that generally extend in the NE direction parallel to the dykes.

The Finsch kimberlite pipe therefore fits the pipe growth model described in this document, with a local stress regime having $\sigma_1$ orientated approximately NE–SW, and $\sigma_3$ orientated approximately NW–SE.

3.3. Case study 3: The Gross Brukkaros Volcanic Complex

The Gross Brukkaros Volcanic Complex is located about 300 km south of Windhoek in southern Namibia. It is considered part of the Gibeon kimberlite field but consists of a central carbonatitic Gross Brukkaros volcano surrounded by more than 100 dykes and by 74 identified volcanic diatreme vents. All data for this case study is taken from Kurszlaukis (1994) that contains detailed mapping of the complex’s diatremes.

Kurszlaukis (1994) considers a multiple stage model for the genesis of the complex starting with the intrusion of a laccolith and the development of a 10 km wide dome structure above it. The brittle fracturing of the doming crust caused a “triple junction” of fractures to form. Three radial dyke swarms intruded the fractures with approximately 120° between the orientations of each swarm (Fig. 11). Vent diatremes formed on the dykes.

The stress conditions during doming are summarized by Stachel et al. (1994) and Kurszlaukis (1994) who consider the greatest vector of compression to be perpendicular to the ground surface, the intermediate vector of compression radial to the dome/volcanic centre and the least vector of compression concentric to the volcanic centre. In simplification to plan view plane-strain conditions in Fig. 11 the radial principal vector is labelled $\sigma_1$ and the concentric principal vector $\sigma_2$. Kurszlaukis notes the elongated shape of the vent diatremes with many vent long axes orientated concentric to the volcanic centre.

![Fig. 11. Plan view of the Gross Brukkaros volcanic field, with the main volcanic crater sediments in the centre of the figure and surrounded by dykes and volcanic vents. Modified after Kurszlaukis (1994). An early phase of uplift produced the “shadowed” triple junction of radial dyke swarms. The interpreted centre of doming is encircled. Labelled arrows indicate the expected stress distribution during uplift with $\sigma_1$ parallel to the dykes. The inset graph shows a frequency plot of vent pipe elongations relative to the stress field. The most frequent shape orientations are parallel and perpendicular to $\sigma_1$.](image-url)
(Fig. 11, graph inset), i.e. perpendicular to \( \sigma_1 \). The inset in Fig. 11 also illustrates that many vent elongations are also orientated parallel to \( \sigma_1 \).

Careful consideration of the vent pipe geometries suggests that the pipes elongated radial to the volcanic centre more commonly pinch and swell in width multiple times along the trend of the feeder dyke and have a high overall aspect ratio (length/width). This can easily be explained by multiple blows or volcanic explosions on the dyke. The blows coalesce to form a larger vent. Kurszlaukis (1994) considers the eruption mechanisms to be dominantly phreatomagmatic. Pipes that are elongated concentric to the volcanic centre typically have a smaller aspect ratio. Some pipes are observed to have curved fractures in the country rock within a few tens of centimeters from the pipe walls and orientated parallel to the pipe walls. These “slabs” of country rock could have formed by thermal spalling or stress-induced fracturing, but the latter seems more likely (Fig. 1).

It is conceivable that the concentrically elongated pipes formed on concentric ring faults formed around the complex during a latter period of magma depressurization and chamber collapse (Kurszlaukis, 1994), but no such structures were mapped and it has been noted that the vent diatremes appear to be coeval with the feeder dykes on which they are located. However, the pipe growth model presented in this paper neatly explains the shapes of the concentrically elongated vent diatremes as growing perpendicular the maximum vector of compression in plan view, and suggests that stress-induced scaling or slabbing of jointed country rock dominated the growth of many of the pipes.

4. Discussion and conclusions

Stress-induced slabbing of the jointed country rock around a volcanic pipe can be an important pipe growth mechanism. The UDEC numerical modelling experiments confirm rock mechanics theory that predicts that scaling and slabbing will occur on the sides of an excavation 90° to the principal stress of greatest compression (\( \sigma_1 \)). Volcanic pipe growth dominated by slabbing is therefore predicted to be preferentially perpendicular to \( \sigma_1 \). The exact direction of growth may be locally affected by the orientation of pre-existing structures. It is demonstrated above that the model is applicable in the Limpopo Belt, at Finsch Mine and for Gross Brukkaros.

The magnitude of imposed tectonic stress on a pipe does not have to be high in order produce stress-induced scaling or slabbing. With increasing depth below surface the horizontal stress around a pipe increases as a function of overburden weight and Poisson’s Ratio, typically by 9 to 10 MPa per kilometer depth. There will therefore be a critical depth at which stress-induced pipe growth will become apparent, a phenomenon seen in vertical shafts and ore passes in the mining industry (e.g. Menzies, 2004; Joughin and Stacy, 2004). The maximum tangential stress
around a circular excavation in a biaxial stress field is
\[3\sigma_1 - \sigma_2 = p.\] In a pipe with a magma underpressure, the increase in stress on the pipe boundary would be about 20 MPa per kilometer depth. Fig. 12 shows how different tectonic stress ratios \(\sigma_1/\sigma_2\) can influence the maximum stress expected around a circular pipe with minimal internal magma pressure. Experience in the mining industry shows that fracture propagation and scaling can occur at a stress magnitude that is roughly two thirds of the uniaxial compressive strength of the rock (see theoretical calculation of Dyskin and Germanovich, 1993).

Depending on the spatial density and tensile strength of the jointing, a pipe would start to undergo stress-induced slabbing, with failure of pre-jointed blocks, at even lower stresses and shallower depths than scaling. These expected depths and stresses are not quantified in this study because of the limitations of the numerical modeling of the slabbing process undertaken. Only the expected direction of slabbing was modeled. Joints were assumed to be continuous and completely interconnected through the model. In reality joint sets vary in orientation (dip and strike), length and spacing. Far more complex three-dimensional models that include joint propagation through intact rock would be required to properly quantify the slabbing process.

The Finsch pipe was emplaced during a poorly constrained 10–15 Ma period (ca. 125 to 114 Ma) in which kimberlite dyke occurrences in southern Africa show a strong NE–SW trend, where prior to and after this period the most dominant trend is WNW–ESE. The dyke trend can be explained as a result of NE–SW-oriented \(\sigma_1\) stress at the time. The reason for this stress orientation is not clear, but might be related to young ridge push forces, orogenic stresses, and in this discussion assumed to be 10–15 MPa. The critical depth at which stress-induced scaling/slabbing began at Finsch appears to have been around 430 m below the current surface since the pipe elongation only becomes obvious at that depth. The weakest rock in that part of the local stratigraphy is graphitic limestone with a uniaxial compressive strength of ca. 100 MPa (Barnett and Preece, 2002). Scaling would then be expected at a stress of 60–70 MPa and is marked with a grey line on Fig. 12. Unfortunately the depth of erosion of the pipe is not known. However, with the above assumed tectonic stress tensor and rock strength, Fig. 12 would indicate that the critical depth is likely to have been at intermediate “mining” depths of 1000 to 2500 m below the original surface.

The Limpopo Belt pipe examples would appear to be a case where the controlling compressive stresses are far-field stresses caused by orogenic processes on the opposite side of the Zimbabwe craton. At the time of kimberlite emplacement (ca. 519 Ma) the Limpopo Belt could have been reactivated in transpression as it had been at least once previously in the Early Proterozoic (e.g. Holzer et al., 1999). Orogenic compressive strength is assumed in this discussion to be around 25–50 MPa (a typical range in modern compressional settings based on the World Stress Map of Reinecker et al., 2004). The high density of structures of various orientations (Kurszlaukis and Barnett, 2003) present in the repeatedly reactivated Limpopo Belt produces a very low rock mass strength (50–100 MPa). Under such stress and rock strength conditions the stress-induced slabbing growth process can be expected at shallow depths within a few hundred metres below surface (Fig. 12). Kurszlaukis and Barnett (2003) do suggest that only a few hundred meters have been eroded from the Venetia pipes based on the presence of crater facies in some of the pipes.

The Gross Brukkaros example is a case where local topographic stresses due to volcanic doming appear to have influenced the shape of satellite diatremes within the volcanic complex. Many diatremes are elongated perpendicular to the local principal compressive stress in plan view, a geometry that is best explained as a result of stress-induced slabbing. Many other diatremes are elongated along dykes, apparently a result of multiple blows coalescing into one pipe. Lorenz and Kurszlaukis (1997) suggest that a maximum of 550 m of strata was eroded since emplacement of the volcanic field, which suggests that local stresses enhanced by doming must have been close to that expected in compressional tectonic settings (>25 MPa) in order for stress-induced slabbing/scaling to have occurred (Fig. 12). The specific timing of evolution of the volcanic complex therefore influenced the diatreme shapes, i.e. doming prior to satellite diatreme formation.

Following the principals of rock mechanics and the results of the studies presented above, it is suggested that:

1. Pipes in a higher differential stress \(\sigma_1 - \sigma_2\) and higher compressive stress magnitude (locally or tectonically induced) will have a stronger tendency to grow perpendicular to \(\sigma_1\).

2. Pipe slabbing would utilize pre-existing structures at relatively low stresses, but could also involve intact rock fracturing (and rock bursts) at higher compressive stresses, i.e. above approximately two thirds of the uniaxial compressive rock strength.
(3) One or two directions of strong structural weaknesses (joint sets or faults) in the surrounding country rock may significantly influence the stress-induced pipe growth direction away from being perpendicular to $\sigma_1$, but by no more than 45°.

(4) A pipe surrounded by country rock with a greater number of joint sets (3 or more sub-vertical sets each of different average strike) is more likely to grow closer to perpendicular to $\sigma_1$.

(5) High internal magma pressures will deter stress-induced slabbing or scaling, but high internal erosion processes (e.g. turbulence and abrasion) will improve the effectiveness of stress-induced pipe growth.

(6) Volcanic pipes with higher groundwater influx are more likely to incur phreatomagmatic eruptions with explosive pipe growth parallel to $\sigma_1$.

(7) Larger pipes can form from overlapping smaller pipes or eruption centers. Pipes or vents that form on feeder dykes by temporal and spatial migration of the eruption centre along the dyke will develop an early pipe elongation parallel to the dyke. This orientation tends to be parallel to $\sigma_1$, depending on the interaction with structures (Delaney et al., 1986; Pollard, 1987).

An important application of this pipe growth model is the ability to determine from the kimberlite pipe and dyke shapes the paleostress directions during kimberlite emplacement. Careful dating of intrusions combined with geometrical studies as illustrated in this document should elucidate the spatial and temporal associations of kimberlite magmatism with tectonic changes. The temporal and spatial variation in paleostress will greatly benefit tectonic models (e.g. Jelsma et al., 2004) and kimberlite exploration models. In polyvolcanic systems that last over extended periods, stress rotations may be preserved by changes in the shape and growth direction of a single volcanic system.

Further numerical UDEC modelling must be attempted to model volcanic pipes undergoing extensional or transtensional stress during emplacement, or pipes growing in close proximity to large, dominant structural features. Strain accommodated on such large structures, and the resulting stress concentrations and stress shadows on the jogs of such structures would significantly affect pipe growth.

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