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Reliability-Based Design of an Anchor-Supported Excavation

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Abstract. The design of the support of a 10m-deep urban excavation in Argentina is presented. A conventional approach was employed for the base case, where a set of geotechnical parameters is selected based on "engineering judgment" and a support system based on concrete wall and anchors was selected for the excavation. To enhance the understanding of the problem, a reliability-based analysis was performed using Plaxis: numerical models were employed to compute factors of safety and displacements for a range of input parameters and a First Order Second Moment (FOSM) technique was used to evaluate the probability of reaching a limit state. A Python code was developed to integrate FOSM into numerical analysis, resulting in an automatic calculation of the probability of failure using Plaxis. The probability of failure was estimated between 0.2% to 2% for the ultimate limit state and less than 0.8% for the serviceability limit state.

Keywords. Finite element method, reliability analysis, first order second moment.

1. Introduction

The design of anchor-supported excavations employing deterministic finite-element models is now routine engineering in major urban developments in Argentina. The usual procedure is to perform a conventional geotechnical investigation using SPT testing, to determine a reasonable set of material parameters largely based on previous experience, and to design for serviceability employing FEM. Little attention is usually paid to the uncertainties of the most relevant parameters and other design decisions like the depth of the excavation, loads acting in the perimeter, etcetera.

The concept of risk and the use of non-deterministic design methods have been recently brought to the attention of the Argentinian geotechnical engineering community because of the adoption of LRFD procedures by structural engineers. Us geotechnical engineers claim that risk is somewhat controlled by our "cautious estimate of input parameters" but usually provide little support to these statements.

In this paper, a conventional design employing FEM is applied to a 10-m deep excavation in Rosario, Argentina, and the opportunity was taken to perform some probabilistic analyses as well. Probabilistic analyses require multiple runs where parameters change according to some calculation strategy. A Python routine was developed to produce these multiple runs automatically, a procedure which effectively

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allows for the integration of probabilistic calculations to routine FEM analysis and design. The paper describes the process and discusses the value it adds to supporting our "cautious estimates" and to improving our routine design procedures for excavations.

2. Project and site conditions

The project is a 9.6 m deep excavation supported by four rows of passive bars, post-grouted ground anchors and a 20 cm concrete wall in the city of Rosario, Argentina.

The geotechnical profile is composed of 6 m of Upper Pampeano and 15 m of Middle Pampeano overlying dense Puelche sands. The Pampeano Formation is a modified Loess, overconsolidated by desiccation and cemented with calcium carbonate in nodule and matrix impregnation form. The reader is referred to refs. [1-7] for a detailed description and mechanical characterization of this soil. The water table is located below the bottom of the excavation, and the Puelche sands are too deep to have any material influence in the behavior of the excavation or its support system.

3. Finite-element analysis

Analyses were performed employing the HS-Small constitutive model in Plaxis [8]. Relevant material parameters, estimated from SPT results and experience [7, 9-11] are summarized in Table 1. Parameters of the Puelche sands and non-relevant parameters are omitted for brevity, see [8] for the definition of the parameters shown in Table 1. Details of the model are shown in Figure 1. Fully drained conditions and a horizontal phreatic surface located under the excavation were assumed in the analysis.

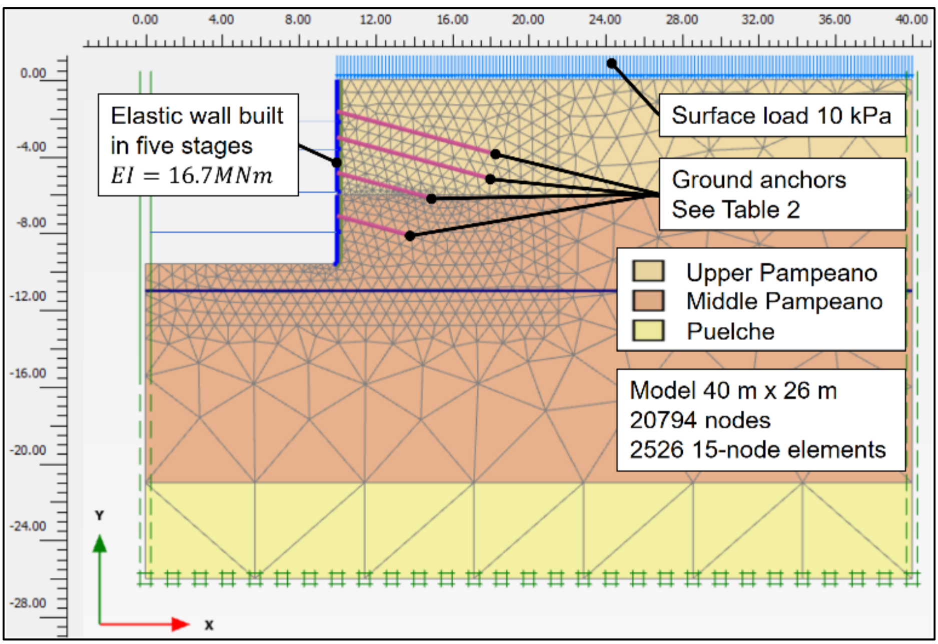

Figure 1. Geometry and finite element mesh.

Table 1. Geotechnical parameters.

Parameter	**Symbol**	**Units**	**Upper Pampeano**	**Middle Pampeano**
Friction angle	ϕ'	º	32	34
Cohesion	c'	kPa	15	30
Dilatancy	ψ	º	0	0
Low strain shear modulus	G_0^{ref}	MPa	200	400
Unload/reload Young's modulus	E_{ur}^{ref}	MPa	120	260
Secant Young's modulus at 50% yield stress	E_{50}^{ref}	MPa	60	110
Oedometer Young's modulus	E_{oed}^{ref}	MPa	60	110
At rest earth pressure coefficient	K_0	-	0.6	0.8

The simulation of the construction process, a standard excavation in five benches, yielded a minimum factor of safety $FoS = 1.40$ at the last stage. Crest displacements of the wall are $U_x = 2.6mm$ (horizontal, to the left in Figure 1) and $U_y = 1.5mm$ (vertical, downwards). Loads in the anchors are shown in Table 2.

Table 2. Characteristics of the ground support system.

Row	**Length**	**Inclination**	**Spacing**	**Yield load**	**Max. Load**
	m	**º**	**m**	**kN**	**kN**
1	7.0	15	2.5	290	290
2	7.0	15	2.5	504	504
3	6.0	15	2.5	497	497
4	4.0	15	2.5	378	378

4. Probabilistic analysis

A probabilistic analysis was performed using two First Order Second Moment (FOSM) methods, namely the Taylor Series Method (TSM) and the Point Estimate Method (PEM) [12-18, 20]. These methods make assumptions on the shape of the target functions like factor of safety and displacements and provide an estimate of their statistical mean and standard deviation. FOSM methods generally require about 100 times less calculation effort than an equivalent Monte Carlo analysis and are therefore most convenient when combined with numerical models like the case presented here.

4.1. Taylor series method

The Taylor Series method (TSM) is based on a Taylor series expansion of the target function considering only two moments (mean μ and standard deviation σ) are considered. The steps to perform the calculation are:

1. Determine the mean μ_i and std. dev. σ_i of the parameters involved.
2. Compute the target function for the mean values $F_{MLV} = F(\mu_1, \dots, \mu_N)$.
3. Compute the standard deviation as $\sigma_F = \sqrt{\Delta F^T \cdot \rho \cdot \Delta F}$, where ρ is the correlation coefficient matrix and ΔF is a vector with elements $\Delta F_i = \big(F(\mu_1, \dots, \mu_i + \sigma_i, \dots, \mu_N) - F(\mu_1, \dots, \mu_i - \sigma_i, \dots, \mu_N)\big)/2$.
4. Define a failure criterion F_f for the target function (e.g. $FoS < 1.0$).
5. Assume a probability distribution for the target function (typically normal and/or lognormal) and compute the probability of failure $PoF = P(F < F_f)$.

The correlation coefficients ρ_{ij} of matrix ρ quantify the degree of linear correlation between variables x_i and x_j, where $\rho_{ij} = 1.0$ means perfectly linear correlation, $\rho_{ij} = -1.0$ means perfectly negative linear correlation, and $\rho_{ij} = 0.0$ means x_i and x_j are uncorrelated.

When using the TSM method it is possible to calculate the absolute sensitivity score of μ_i as $\eta_{SS,i} = \left|\left(F(\mu_1, \dots, \mu_i + \sigma_i, \dots, \mu_N) - F(\mu_1, \dots, \mu_i + \sigma_i, \dots, \mu_N)\right)\right|$ [8]. For N variations, the relative sensitivity score for each parameter is calculated as $x_{i,score} = \eta_{SS,i} / \sum_{i=1}^{N} \eta_{SS,i}$. The procedure requires the target function to be computed $q = 2N + 1$ times, where N is the number of variables of the model.

4.2. Point estimate method

In the point estimate method (PEM) the target function is evaluated 2^N times, combining all parameters evaluated at $\mu_i \pm \sigma_i$, i.e. one standard deviation above and below the mean value. The steps are as follows [15]:

1. Determine the mean μ_i and std. dev. σ_i of the parameters involved.
2. Evaluate the target function for all the permutations of $\mu_i \pm \sigma_i$ ($q = 2^N$ times).
3. Compute the mean value of the target function as $F_\mu = \left(\sum_{i=1}^{q} p_i\, F_i\right)/q$, where $p_i = 1 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} s_i\, s_j \rho_{ij}$, $s_i = +1$ if $x_i = \mu_i + \sigma_i$, $s_i = -1$ if $x_i = \mu_i - \sigma_i$.
4. Compute the standard deviation $\sigma_F = \left(\sum_{i=1}^{q} p_i F_i^2 - F_\mu^2\right)^{1/2}$.
5. Steps 5 and 6 are equal to those of the Taylor Series Method.

For example, in the case of two correlated variables, the mean value μ_F and standard deviation σ_F of the target function are calculated from the following evaluations: $F_1 = F(\mu_1 + \sigma_1,\ \mu_2 + \sigma_2)$, $F_2 = F(\mu_1 - \sigma_1,\ \mu_2 + \sigma_2)$, $F_3 = F(\mu_1 + \sigma_1,\ \mu_2 - \sigma_2)$ and $F_4 = F(\mu_1 - \sigma_1,\ \mu_2 - \sigma_2)$ and the weighing coefficients $\rho_1 = 0.25(1 + \rho_{12})$, $\rho_2 = 0.25(1 - \rho_{12})$, $\rho_3 = 0.25(1 - \rho_{12})$, and $\rho_4 = 0.25(1 + \rho_{12})$. It is noted that μ_F is calculated as the weighted average of all F_i.

PEM procedure yields different results than the TSM, where the mean value of the target function is calculated by evaluating the target function using the mean value of the parameters [19].

4.3. Implementation

A Python code was developed to control Plaxis as follows: i) to perform permutations of the selected values of the geotechnical parameters; ii) to create, run and store realizations of the finite-element model using the adopted values of the variables; iii) to extract numerical evaluations of the selected target functions (FoS, U_x, U_y); iv) to compute the probability of failure, defined as the probability of the result not meeting the relevant limit state. Aleatory variables and their coefficient of variation $c_V = \sigma/\mu$ are shown in Table 3. The four parameters that control stiffness (E_{ur}, E_{50}, E_{oed}, G_0) were treated as a single aleatory variable in the computations, i.e. were varied at the same time and in the same direction ($\pm\sigma$).

Table 3. Geotechnical parameters adopted as aleatory variables.

Parameter	Symbol	Coefficient of Variation	
		Upper Pampeano	Middle Pampeano
Cohesion	c'	50.0	50.0
Friction angle	ϕ'	9.4	8.8
Stiffness parameters	E_{ur}^{ref}, E_{50}^{ref}, E_{oed}^{ref}, G_0^{ref}	20.0	20.0

The geometry, the thickness of layers and the structural strength of the anchors were considered deterministic. Coefficients of variation were selected using the "N-sigma rule" method [17]: extreme high and low values of the variables were estimated based on engineering judgment and the standard deviation was then estimated as 25% of the difference between the extremes estimated for each variable. Two correlation coefficients were considered between the cohesion and friction angle of each layer: $\rho_{\phi|c} = -0.5$ and $\rho_{\phi|c} = 0.0$ (i.e. uncorrelated) for both layers simultaneously. Three limit states were considered, namely $FoS < 1.0$, horizontal displacement at the crest $U_x > 10\ mm$, and settlement at the crest $U_y > 10\ mm$. Two statistical distributions, normal and log-normal, were selected for the shape of each target function.

The exercise was completed with a test for robustness of the procedure, where the Upper Pampeano layer was subdivided in two sublayers of equal thickness and two sets of calculations using both PEM and TSM procedures were performed. In the first set, parameters of the two sublayers were treated as a single aleatory variable, i.e, only one thick layer was considered. In the other set, the parameters of the two sublayers were adopted as independent aleatory variables with no correlation between the layer, effective treating the sublayers as completely independent materials.

Thirteen runs were required for the TSM analysis and sixty-four for the PEM method in the scenario where the Upper Pampeano was evaluated as one layer. Nineteen runs were required for the TSM analysis and five hundred and twelve for the PEM method in the scenario where the two sublayers of the Upper Pampeano where evaluated independently. It is worth noting that there are no common models between the PEM and the TSM methods, so this exercise required seventy-three and five hundred thirty-one runs for each scenario.

4.4. Results

Table 4 shows the mean and standard deviation values calculated for all the target functions with TSM and PEM. The results labeled as "one layer" refer to the scenario where the two sublayers of the Upper Pampeano were evaluated as a single layer, while "two layers" refers to the scenario where they were treated as different materials.

Table 4. Mean and standard deviation of the target functions.

Method	$\rho_{\phi\|c}$	FoS		U_x (mm)		U_y (mm)	
		μ	σ	μ	σ	μ	σ
PEM one layer	0.0	1.410	0.205	3.755	1.783	2.652	1.801
PEM one layer	-0.5	1.412	0.168	3.385	1.283	2.298	1.358
TSM one layer	0.0	1.428	0.214	2.590	1.226	1.497	1.356
TSM one layer	-0.5	1.428	0.178	2.590	1.099	1.497	1.198
PEM two layers	0.0	1.408	0.200	3.719	1.600	2.573	1.615
PEM two layers	-0.5	1.410	0.164	3.376	1.168	2.249	1.221
TSM two layers	0.0	1.425	0.209	2.548	1.148	1.453	1.246
TSM two layers	-0.5	1.425	0.168	2.548	1.027	1.453	1.098

The number of digits reported in the table is just to show the scatter of the procedure; it should not be taken as an indicator of the claimed precision of the results. For instance, the difference between the mean values of the target functions for the "one layer" and "two layers"cases varies between 0.1% to 3.0%. At the same time, the difference between the standard deviation of the same scenarios varies between 2% to 10%.

Table 5 shows the probability of failure for both PEM and TSM, for all the analyzed limit states. It is worth noting that the ratio of the maximum PoF (considering PEM and TSM) for "one layer" and "two layers"cases is 1.5 and 2.2 for the normal and lognormal distribution respectively.

Figure 2 shows the probability distribution and cumulative probability function of U_y using PEM, for uncorrelated cohesion and friction angle, and the "one layer" case. It is noted that the lognormal distribution does not admit negative values for U_y, whether the normal distributions allows for the probability of vertical displacements of the crest of the retaining wall to be in the upward direction (i.e. a negative value), a result that is consistent with a purely elastic rebound and should in principle be admitted by the procedure.

Table 5. Probability of failure for the three target functions.

Method	$\rho_{\phi\|c}$	$P[FoS < 1.0]$		$P[U_x > 10mm]$		$P[U_y > 10mm]$	
		normal	lognormal	normal	lognormal	normal	lognormal
PEM one layer	0.0	7E-03	2E-03	1E-07	8E-04	7E-09	2E-03
PEM one layer	-0.5	2E-02	1E-02	2E-04	8E-03	2E-05	7E-03
TSM one layer	0.0	2E-02	1E-02	7E-10	6E-04	2E-10	2E-03
TSM one layer	-0.5	8E-03	3E-03	8E-12	2E-04	6E-13	1E-03
PEM two layers	0.0	2E-02	9E-03	4E-05	5E-03	2E-06	4E-03
PEM two layers	-0.5	6E-03	2E-03	7E-09	3E-04	1E-10	7E-04
TSM two layers	0.0	2E-02	9E-03	4E-11	3E-04	3E-12	1E-03
TSM two layers	-0.5	6E-03	2E-03	2E-13	1E-04	4E-15	7E-04

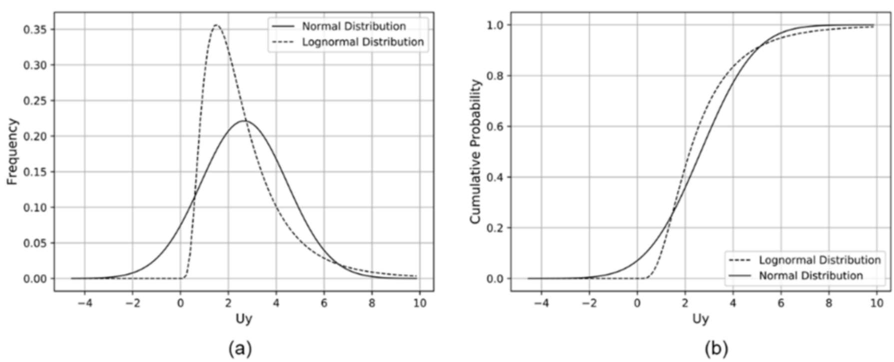

Figure 2. Distribution of probability (a) and cumulative probability (b) of the vertical displacement U_y for uncorrelated cohesion and friction angle using PEM and both normal and lognormal distributions.

Using the results from the TSM, the sensitivity score of each variable was calculated for the three target functions. These results are shown in Figure 3, for the "one layer" case. As expected, cohesion is the parameter that most influence the three target functions. FoS is most affected by the cohesion of the Middle Pampeano, the material that forms the lower half of the excavated wall (see Figure 1). The vertical and horizontal displacements are equally affected by the cohesion of both layers. Also, the figure shows that stiffness parameters have much less impact on the uncertainty of results, a known condition for FoS usually employed to support the use of simple Mohr-Coulomb plasticity type models for ultimate limit state (safety) analyses, but that can be extended in this analysis to the serviceability limit state.

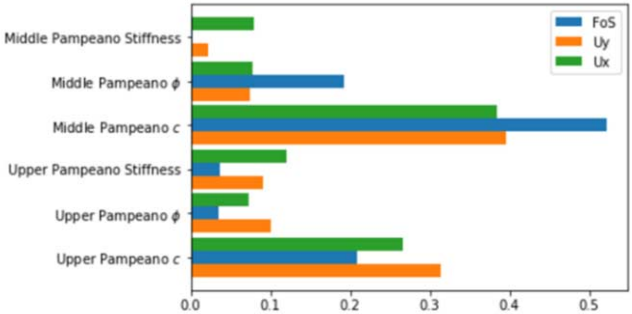

Figure 3. Sensitivity score plot for all the variables and the different target functions: FoS, vertical displacement U_y and horizontal displacement U_x.

4.5. Analysis of results

Given the strong nonlinearity of the limit state functions, the FOSM methods used for the calculations only provide an estimate of the probability of failure. That is to say, the results must be analyzed in terms of order of magnitude of PoF. In this regard, PoF of 0.1%, 1.0% or 10% can be viewed as low, medium, and high [17], respectively. In this context, and for this example, PoF of the ultimate limit state (i.e. $P[FoS < 1.0]$) is within the low to medium range (between 2% and 0.2%). PoF in terms of serviceability limit state analysis (i.e. vertical and horizontal displacements less than 10mm) is low (less than 1.0%).

It is observed that TSM presents advantages when compared to the PEM worth noting: i) it requires fewer computations (2N+1 instead of 2^N); and ii) it provides some insight on which are the input parameters that have a stronger influence in the results. In the PEM, as all the input parameters are varied at the same time, it is difficult to isolate and address the impact of any individual parameter on the results of the analysis.

Regarding accuracy, it is not possible to judge which one gives the best estimation of PoF without resourcing to a numerically very expensive Monte Carlo analysis requiring hundreds of thousands of runs for these low-probabilty design case. As both TSM and PEM methods provide reasonable estimates of PoF at an affordable cost, there is value in doing both analyses. Starting the analysis with TSM is recommended, because it highlights which parameters are worth varying in the more expensive PEM analysis, thus providing the opportunity to reduce the number of aleatory variables of the latter.

5. Conclusions

A numerically based reliability analysis on the design of the anchor support system for a 10m-depth urban excavation in Argentina was presented in this work. Two First Order Second Moment (FOSM) methods, the Taylor Series Method (TSM) and the Point Estimate Method (PEM), were employed to estimate the probability of failure of the ultimate limit state (i.e. $P[FoS < 1.0]$) and for serviceability limit state (i.e. probability of vertical and horizontal displacements being larger than 10 mm). These evaluations required 64 runs of the FEM model for the PEM and 13 times for the TSM. These calculations were performed using a Python code that operated Plaxis automatically, extracted and stored the relevant results.

The probability of loss of stability was evaluated to be between 2% to 0.2%; the probability of loss of serviceability was evaluated to be less than 0.8%. A discussion was presented regarding the advantages and limitations of each method.

The robustness of the procedure was tested by dividing one layer into two sublayers and evaluating the response of the system by treating the matching parameters of the two sublayers both as a single aleatory variable and as two independent variables. This resulted in an additional 512 calculations for the PEM and 19 runs for the TSM.Small changes of the mean and standard deviation of the target functions were observed, up to 3% and up to 10% respectively. However, these differences result in a variation of the probability of failure of almost one order of magnitude for one of the target functions, thus proving that the division of the ground profile in layers, a task that is still done manually and employing “engineering judgement” is by itself a source of uncertainty that cannot be overlooked when performing reliability-based design.

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